Assignment # 1 (Unit 1)

Engineering Mathematics-I (TMA-101)

1. Define rank of a matrix.
2. Reduce the given matrices to normal form and hence find rank.

(i) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 6 & -1 & -9 & -17 \end{bmatrix}$

3. Test the consistency and hence solve the following system of equations
$x+2y+z=2,\ \ 3x+y-2z=1,\ \ 4x-3y-z=3$ and $2x+4y+2z=4.$
4. Find the values of k, such that the system of equations has non-trivial solution.
$$4x+9y+z=0,\ \ kx+3y+kz=0,\ \ \ x+4y+2z=0.$$
5. Solve the following system of linear equations:
$2x_1+x_2+2x_3+\ x_4=6,\ \ 6x_1-6x_2+6x_3+12x_4=36,\ \ 4x_1+3x_2+3x_3-3x_4=-1,$

$2x_1+2x_2-x_3+\ x_4=10$ Ans: $x_1=2,\ x_2=1\ \ x_3=-1,\ x_4=3.$

6. Show that the system of linear equations
3x+3y+2z=1, x+2y=4, 10y+3z=-2 and 2x-3y-z=5 is consistent and
hence find the solution. Ans: x=2, y=1 and z=-4.

7. Investigate for what values of λ and μ do the system of equations
x + y + z=6, x+2y+3z=10, **x**+2y+ λ z= μ have
(i) No solution (ii) Unique solution (iii) Infinite solutions?
Ans: (i) λ=3, μ≠10 (ii) λ≠3 (iii) λ=3, μ=10.
8. For what values of λ and μ, the following system of equation
$5x+3y+2z=9,\ \ \text{-}2x+3y+7z=8,\ \ \lambda x+3y+2z=\mu$
will have (i) unique solution (ii) infinite no. of solutions (iii) no solution.
9. Show that the system of equations;
$-2x+y+z=a,\ \ x-2y+z=b,\ \ x+y-2z=c,$
has no solution unless $a+b+c=0$. In which case there are infinitely many solutions?
10. Following vectors are linearly dependent or independent, if dependent find the relation between them
(i) (2,-1,0,5), (3,1,7,-1), (2,0,-3,4)
(ii) $X_1=(2,0,1)$, $X_2=$ (-3,5,0), $X_3=$ (4,0,2), $X_4=$ (3,2,1)
(iii) $X_1=(1,-1,1)$, $X_2=$ (-3,2,0)

11. Find the Eigen values , Eigen vectors of the following matrices

$$(i)\begin{bmatrix}1 & 1 & 3\\1 & 5 & 4\\3 & 4 & 2\end{bmatrix} \qquad (ii)\begin{bmatrix}-2 & 1 & 1\\-11 & 4 & 5\\-1 & 1 & 0\end{bmatrix}$$

12. Find the characteristic equation of the following matrices and verify Caley-Hamilton Theorem and hence find the inverse

$$(i)\begin{bmatrix}5 & -1 & 5\\0 & 2 & 0\\-5 & 3 & -15\end{bmatrix}$$

13. Find the characteristic equation of the matrix A= $\begin{bmatrix}2 & 1 & 1\\0 & 1 & 0\\1 & 1 & 2\end{bmatrix}$ and hence compute A^{-1}.

Also find the matrix represented by A^8-5 A^7+7 A^6-3 A^5+ A^4-5 A^3+8 A^2-2A+I.

Ans: λ^3-5 λ^2+7λ-3=0, $A^{-1}=\frac{1}{3}\begin{bmatrix}2 & -1 & -1\\0 & 3 & 0\\-1 & -1 & 2\end{bmatrix}$, $\begin{bmatrix}8 & 5 & 5\\0 & 3 & 0\\5 & 5 & 8\end{bmatrix}$

14. Show that the matrix A has repeated eigen values. Also find the corresponding eigen vectors,

where A= $\begin{bmatrix}2 & 0 & 2\\-1 & 3 & 1\\1 & -1 & 3\end{bmatrix}$ **Ans:** λ=2, 2, 4; $\begin{bmatrix}1\\1\\0\end{bmatrix}$, $\begin{bmatrix}1\\0\\-1\end{bmatrix}$

15. Reduce the matrix A= $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -1\\0 & 0 & 3\end{bmatrix}$ to diagonal form by similarity transformation.

Hence find A^3. **Ans:** D = $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$, $A^3=\begin{bmatrix}1 & -7 & 32\\0 & 8 & 19\\0 & 0 & 27\end{bmatrix}$

16. Prove that the given matrix is unitary: $\begin{bmatrix}\frac{1}{2}(1+i) & \frac{1}{2}(-1+i)\\\frac{1}{2}(1+i) & \frac{1}{2}(1-i)\end{bmatrix}$

17. Show that the matrix $\begin{bmatrix}\alpha+i\gamma & -\beta+i\delta\\\beta+i\delta & \alpha-i\gamma\end{bmatrix}$ is unitary if and only if $\alpha^2+\beta^2+\gamma^2+\delta^2$=1.